MA 3046 - Matrix Algebra
Prerequisite Skills

The student wishing to take this course should possess certain minimal skills, in order to have a reasonable expectation for successful completion. Specifically, they should be able to:

1. Define the terms scalar, matrix, row vector and column vector. Identify specified elements, rows and columns in a given matrix.

2. State and apply the rules for multiplication of a matrix or vector by a scalar, for the addition and subtraction of two compatible matrices or vectors, and for the multiplication of two compatible matrices. Correctly identify and explain under what conditions certain matrix operations cannot be performed. Identify which of the standard properties of the algebra of real numbers do and do not apply to matrix operations.

3. Define the term linear combination of an arbitrary number of row or column vectors. Describe the relationships between linear combination and matrix multiplication. Be able to find a given vector as a linear combination of an appropriate set of other vectors.

4. Define the terms identity matrix, matrix transpose, symmetric matrix and matrix inverse. Determine whether a given matrix is symmetric or not, and, if not, find its transpose.

5. Define what is meant by a partitioned (or block) matrix, and under what conditions partitioned matrices can be added and multiplied. Given a matrix, express it in at least one valid partitioned form. When appropriate, calculate the result of addition or multiplication of partitioned matrices.

6. Describe what is meant by a system of linear equations, and correctly distinguish between given linear and nonlinear systems. Express a given system of linear equations equivalently in terms of matrix-vector products and augmented matrices, and vice versa. Define what is meant by the term equivalent system of linear equations.

7. Define the terms upper or lower triangular matrix, and explain why solving linear systems involving either of these matrices is relatively easy.

8. Describe the process of Gaussian elimination elimination, and the roles of elementary row operations, row equivalence, pivot elements, free variables, backward substitution, row echelon form, and row-reduced echelon form. Describe the three possible qualitative outcomes of Gaussian elimination. Use Gaussian elimination to determine whether a given system has any solutions, and, if so, specify them as completely as possible.

9. Define the term homogeneous system of linear equations. Describe how the solutions to such systems differ qualitatively from those of more general, nonhomogenous systems.

10. Define the concept of an Elementary Matrix. Describe the relationship(s) between Gaussian elimination and elementary matrices.

11. State five equivalent conditions that guarantee that a given matrix $\mathbf{A}$ will have an inverse.

12. Given a matrix, determine whether it is invertible, and, if so, find its inverse using elementary row operations or elementary matrices.

13. Define what is meant by the **LU** decomposition of a matrix. Find the **LU** decomposition of a given matrix $\mathbf{A}$ when row interchange is not required during Gaussian elimination. Use the **LU** factorization to solve a given linear system of equations.

14. Evaluate the determinant of a given matrix by the method of cofactor expansion. State the fundamental algebraic properties of determinants.

15. State the definitions of a vector (linear) space and subspace, and, given appropriate sets, determine whether they satisfy the definitions. Define Euclidean n-space ($\Re^n$). Determine whether a given subset of $\Re^n$ forms a subspace. Interpret subspaces and other appropriate sets in $\Re^2$ and $\Re^3$ geometrically.

16. State the definitions of span, linear independence, basis and dimension. Describe how these terms can be interpreted for vectors in $\Re^2$ and $\Re^3$. Describe also how these are related to the solution of $\mathbf{Ax} = \mathbf{b}$. Describe the standard basis for $\Re^n$.

17. State the definitions of an ordered basis and of the coordinates of a vector relative to an ordered basis. Describe the relationship between the coordinates in different bases. Convert a given vector between its coordinates in terms of the standard basis and its coordinates in terms of a given non-standard basis, or between its coordinates in terms of two given non-standard bases.

18. State the definitions of the row space (the Row Space of $\mathbf{A}$), column space (the Column Space of $\mathbf{A}$) and null space (the Null Space of $\mathbf{A}$) of a general matrix. Describe how these subspaces relate to the existence and uniqueness of solutions to $\mathbf{Ax} = \mathbf{b}$. Describe the relationships between Gaussian elimination, linear independence and bases for the Row Space of $\mathbf{A}$ and the Column Space of $\mathbf{A}$. Use elimination to construct a basis from a spanning set in $\Re^n$.

19. State the definition of the rank of a matrix, and, given an appropriate matrix, find its rank. Describe the relationships between Gaussian elimination, the rank of $\mathbf{A}$ and the solution of $\mathbf{Ax} = \mathbf{b}$. Characterize rank 1 matrices.

20. State the definition of a linear transformation, and apply that definition to determine whether a given transformation is linear or not. Describe geometrically the general behavior in $\Re^3$ of the special cases or linear transformation represented by rotations, reflections and projections.

21. State the definition of the standard representation of a linear transformation and describe the principal underlying that representation. Given a linear transformation, find the matrix representation of that transformation relative to both the standard basis and to any given set of non-standard bases for both the domain and the range. Explain the relationship of the term similarity to matrix representations of linear transformations.

22. Define the standard inner product in $\Re^n$. Describe the relationship between the standard inner product and matrix multiplication. Calculate standard inner products, distances, "angles" and vector norms in $\Re^n$. State the definition of orthogonality in $\Re^n$.

23. State the definition of orthogonal and orthonormal sets, and describe their relationships to coordinates and systems of linear equations.

24. State the definition of the left null space of a matrix (the Null Space of $\mathbf{A}^T$).

25. State the definition of the orthogonal complement of a subspace, and give geometrical interpretations in $\Re^2$ and $\Re^3$. Describe, in terms of orthogonal complements, the relationships between the the four fundamental subspaces of a matrix - i.e. the Row Space of $\mathbf{A}$, the Column Space of $\mathbf{A}$, the Null Space of $\mathbf{A}$ and the Null Space of $\mathbf{A}^T$.

26. State the definitions of and give geometrical interpretations for the terms eigenvalue, eigenvector and eigenspace. Given an appropriate matrix, find all of its eigenvalues, an associated set of eigenvectors, and the dimension of each associated eigenspace.

27. Define what is meant by a diagonalizable matrix, and the relationship of this concept to eigenvalues and eigenvectors. Given a matrix, find a matrix which diagonalizes it, or describe why such a matrix does not exist.

28. Describe the special properties enjoyed by the eigenvalues and eigenvectors of a symmetric matrix.

29. Define the term similar matrices. Explain why similar matrices must have the same eigenvalues and describe the relationship between their eigenvectors.

30. Use MATLAB to create matrices and vectors, and to write simple programs (**m**-files) that include loops, "if" statements, calls to subroutines/subprograms, and simple graphical and numerical outputs.

Reference: *Elementary Linear Algebra*, by Howard Anton -
Chapters 1, 2, 3.1-3.3, 4.1, 5.1-5.3, 6.1-6.2, 7.1-7.2